Magnetic After-Effect. I

Magnetic After-Effect. I*

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(Received January 28, 1956)

Synopsis

The magnetic after-effect was studied from the standpoint of the activation-process, and the fundamental equations somewhat different from others were theoretically obtained. Further, the origin of the retardation of the irreversible displacement of Barkhausen domain wall and the exact relation between the magnetic after-effect and lattice defects were discussed.

I. Introduction

Though studies(1) have been made of the magnetic after-effect in connection with the reversible process of magnetization, the present author(2) observed Barkhausen effect in the region of irreversible magnetization and concluded that the delayed irreversible displacement of a magnetic wall might also be the cause of the after-effect. The magnetic after-effect in the irreversible region was experimentally studied by Maeda(3). By using the alloy Ni_3M_n, he found that the magnetic after-effect could be expressed by

$$\ln J_n(t) = \ln J_{no} - n \ln t \ , \qquad (1)$$

and

$$\ln n = \text{const} - \ln N \ , \qquad (2)$$

where $J_n(t)$ is the total change in magnetization from t seconds after a sudden change in the field to the time at which it becomes practically unobservable, J_{no} the value of $J_n(t)$ for $t = 1$ second and N the demagnetization factor of the specimen. The effective field $H-NJ$ is not constant in the course of the after-effect, since the change in magnetization ΔJ results in the change in the demagnetizing field $N\Delta J$. Hence, if N is large, the effective field will change considerably, however small the change in magnetization may be. Accordingly, $H-N\Delta J$ must always decrease during the magnetic after-effect, the characteristics of which should, then, depend on the dimension of the specimen. Hence, the present investigation was carried out on the magnetic after-effect in the irreversible region from the standpoint of the rate-process and the result was compared with the empirical formulas and the observation.

II. Theoretical consideration

The ferromagnetic material may be assumed to be composed of small blocks, say, magnetic domains. Denote the activation energy for the displacement of a

* The 827th report of the Research Institute for Iron, Steel and Other Metals.

(1) G. Richter, Ann. Phys., **29** (1937), 605.

(2) T. Huzimura, J. Phys. Soc. Jap., **5** (1950), 293.

(3) S. Maeda, J. Phys. Soc. Jap., **6** (1951), 494; Sci. Rep. RITU, **A4** (1952), 376.

magnetic wall through a domain by U, then U will be reduced to $U-qJ_sH$ under the action of the magnetic field H, in which J_s is the saturation value of magnetization and q the volume in which thermal fluctuation takes place. But if the demagnetization factor N is fairly large, H must turn into the effective field, $H_{eff}=H-NJ$, and the activation energy will, accordingly, become $U'=U-qJ_s(H-NJ)$. Therefore, in the course of the magnetic after-effect the activation energy always increases. The number of activated walls can be assumed to be approximately proportional to the inclination of the magnetization curve and, hence, varies with the change in magnetization. In the following calculation, however, the number of walls is assumed to be invariant, for the change in magnetization in the course of the magnetic after-effect is vary small.

Now, the rate of change in magnetization is given by

$$\frac{dJ}{dt}=K\exp\left\{-\frac{U-qJ_s(H-NJ)}{kT}\right\}, \tag{3}$$

in which several factors which can be assumed to be invariant are included in K. By integration, Eq. (3) will become

$$\exp\left\{\frac{qJ_sNJ}{kT}\right\}=\frac{KqJ_sN}{kT}\exp\left\{-\frac{U-qJ_sH}{kT}\right\}t+\text{const.} \tag{4}$$

Denote the magntization immediately after the change of the field by J_0, then Eq. (4) will be turned into

$$\exp\left\{\frac{qJ_sN(J-J_0)}{kT}\right\}=\frac{KqJ_sN}{kT}\exp\left\{-\frac{U-qJ_s(H-NJ_0)}{kT}\right\}t+1, \tag{5}$$

or,

$$J-J_0=\Delta J_t=\frac{kT}{qJ_sN}\ln\left(\frac{t}{t_s}+1\right), \tag{6}$$

$$t_s=\frac{kT}{KqJ_sN}\exp\left\{\frac{U-qJ_s(H-NJ_0)}{kT}\right\}. \tag{7}$$

Eq. (6) is the fundamental equation of the change in magnetization in the course of the magnetic after-effect.

The procedure of measurement by Maeda[3] was as follows: A search coil is wound around the specimen inserted in the magnetizing solenoid and is connected to the ballistic galvanometer. The magnetization of the specimen is first changed by the external field, and after t seconds, the circuit of the galvanometer is closed. The total change in magnitization after t seconds, $J_n(t)$, is measured by the ballistic method. The relation between ΔJ_t and $J_n(t)$ thus obtained is shown in Fig. 1.

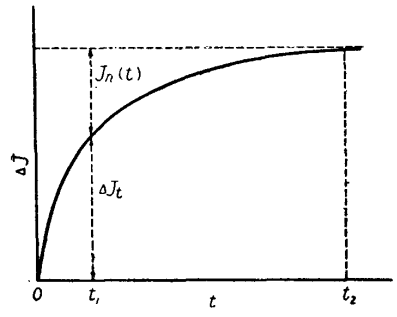

Fig. 1. Relation between $J_n(t)$ and ΔJ_t.

Now, suppose that the circuit is closed at the moment t_1 and opened at t_2, and denote $J_n(t)$ for t_0 by J_{no}, then $J_n(t)$ can be expressed by the following equation:

$$J_n(t_1) = \Delta J_{t_2} - \Delta J_{t_1} = J_{no} - (\Delta J_{t_1} - \Delta J_{t_0}), \tag{8}$$

which, by taking logarithm, will be

$$\ln J_n(t) = \ln\left\{ J_{no}\left(1 - \frac{\Delta J_{t_1} - \Delta J_{t_0}}{J_{no}}\right)\right\} \tag{9}$$

As $J_{no} > \Delta J_{t_1} - \Delta J_{t_0}$, Eq. (9) may be expanded in series. Inserting ΔJ_t (Eq. (6)) in Eq. (9) and neglecting higher order terms, Eq. (9) can be expanded approximately as

$$\ln J_n(t) = \ln J_{no} - n\left\{ \ln\left(\frac{t}{t_s} + 1\right) + c\right\}, \tag{10}$$

where

$$n = \frac{kT}{qJ_s J_{no} N} \tag{11}$$

In discussing Eq. (10) two cases must be considered:

1. $t > t_s$

When the second term in the right side of Eq. (10) is expanded in series of t_s/t, the following formula will be obtained:

$$\ln J_n(t) \fallingdotseq \ln J_{no} - n\left\{ \ln\left(\frac{t}{t_s}\right) + \frac{t_s}{t} + c_1\right\}, \tag{12}$$

where

$$c_1 = -\frac{t_0}{t_s} + \cdots\cdots \qquad \text{for} \quad t_0 < t_s < t$$

$$\ln J_n(t) \fallingdotseq \ln J_{no} - n\left\{ \ln\left(\frac{t}{t_0}\right) + \frac{t_s}{t} + c_2\right\}, \tag{13}$$

where

$$c_2 = -\frac{t_s}{t_0} + \cdots\cdots \qquad \text{for} \quad t_s < t < t_0,\ t_s < t_0 < t$$

If $t \gg t_s$,

$$\ln J_n(t) = \ln J_{no} - n \ln t + \text{const.} \tag{14}$$

2. $t < t_s$

Expanding Eq. (10) in series of t/t_s,

$$\ln J_n(t) \fallingdotseq \ln J_{no} - n\left(\frac{t}{t_s} + c_3\right), \tag{15}$$

where

$$c_3 = -\frac{t_0}{t_s} + \cdots\cdots \qquad \text{for} \quad t_0 < t < t_s,\ t < t_0 < t_s$$

and

$$c_3 = -\frac{t_s}{t_0} + \cdots\cdots \qquad \text{for} \quad t < t_s < t_0$$

Substituting t_s and n by Eq. (7) and (11), respectively,

$$\ln J_n(t) = \ln J_{no} - \frac{K}{J_{no}} t + \text{const.} \tag{16}$$

Further, by taking logarithm of Eq. (11),

$$\ln n = \ln\left(\frac{kT}{qJ_s}\right) - \ln J_{no} - \ln N, \tag{17}$$

and if $\ln(kT/qJ_sN) \gg \ln J_{no}$, the above equation will be reduced to Eq. (2). Thus,

Eqs. (14) and (17) theoretically obtained correspond respectively to the empirical formulas (1) and (2) found by Maeda.

III. Comparison of theory and experiment

The alloys Ni_3M_n was found to show a remarkably large after-effect, and as already mentioned, the measurements were carried out by Maeda in the region of irreversible magnetization with specimens of various dimension ratios, the results of which were expressed by Eqs. (1) and (2). It is necessary, moreover, to bear in mind that Eq. (14) is applicable to the explanation of the experimental results only under the condition $t > t_s$.

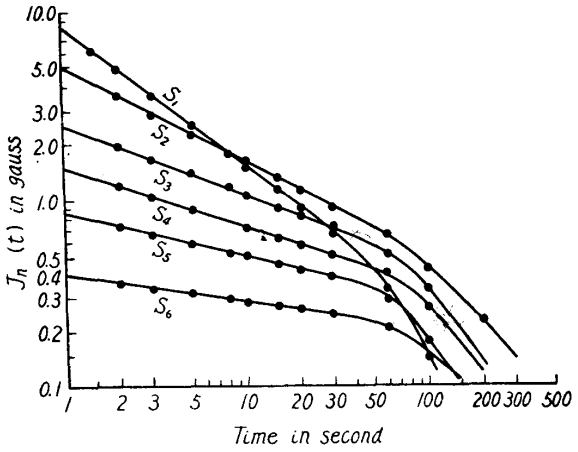

Fig. 2. Curves of $J_n(t)$ vs. t for various values of dimension ratio. (after Maeda)

According to Maeda, the after-effect in Ni_3M_n varied with the dimension ratio as shown in Fig. 2. The fundamental equation of after-effect may be turned into

$$\ln J_n(t) = \ln J_{no} - n \ln\left(\frac{t + t_s}{1 + t_s}\right) \tag{18}$$

Suppose that n is the tangent of the curve $\ln J_n(t)$ versus $\ln t$, and let J_{no} and $J_n(10)$ be the value of $J_n(t)$ at $t=1$ and 10 seconds respectively, then t_s can be computed by Eq. (18), the results of which are shown in Table 1.

Table 1

Sample	N	n	J_{no} (gauss)	$J_n(10)$(gauss)	t_s (sec)	K
S_1	0.0012	0.76	8.3	1.5	1.4	0.80
S_2	0.0044	0.50	5.1	1.6	1.4	0.56
S_3	0.0067	0.36	2.5	1.1	1.4	0.28
S_4	0.015	0.31	1.5	0.72	1.4	0.17
S_5	0.037	0.25	0.86	0.50	1.3	0.01
S_6	0.072	0.18	0.40	0.29	1.3	0.05

As seen in the table, t_s is about 1 second in every case. As the creep of magnetization continues over some hundred seconds, t is always larger than t_s, that is, the above-mentioned condition $t > t_s$ strictly holds good in the case of Ni_3M_n. Substituting the experimental values for n, J_{no} and N, the first term in the right

Table 2

Sample	N	n	$\log(kT/qJ_s)$	$\frac{q}{(J_s = 800 \text{ gauss})}$
S_1	0.0012	0.76	−2.02	$5.43 \cdot 10^{-15}$
S_2	0.0044	0.50	−1.95	4.62
S_3	0.0067	0.36	−2.25	8.60
S_4	0.015	0.31	−2.16	7.48
S_5	0.037	0.25	−2.10	6.52
S_6	0.072	0.18	−2.29	10.3

side of Eq. (17) can be computed and consequently, q will be estimated, the results of which are shown in Table 2. As seen in the table, the respective values of the former are the same as one another within the error of 5 per cent.

One of the important results emphasized by Maeda was the fact that n is independent of temperature, which can be seen in Fig. 3. Take $n = 0.5$ and $N = 0.0044$ as the probable values, then q varies with temperature as shown in Table 3, that is, it increases with the rise of temperature.

The after-effect of technical iron (0.03 per cent C) was not

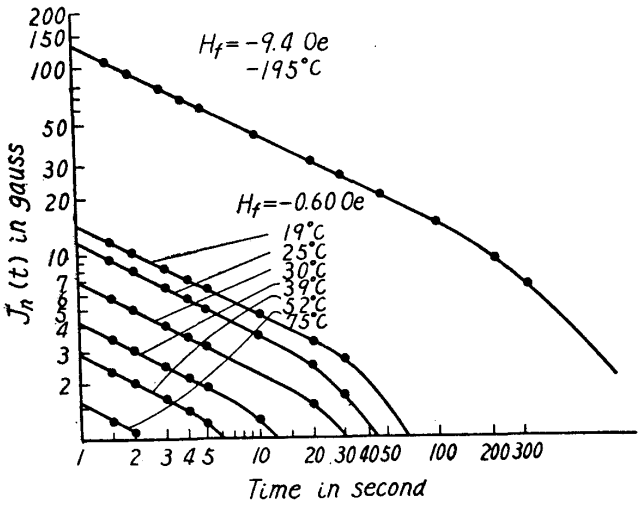

Fig. 3. Curves of $J_n(t)$vs. t at different temperatures. (after Maeda)

Table 3

T (K)	83	292	298	303	312	325	348
J_{no}(gauss)	140	14	12	7	4.2	2.9	1.1
q (cm^3)	$\times 10^{-17}$ 6.1	$\times 10^{-15}$ 2.2	" 2.64	" 4.4	" 7.2	" 11.9	" 32.6

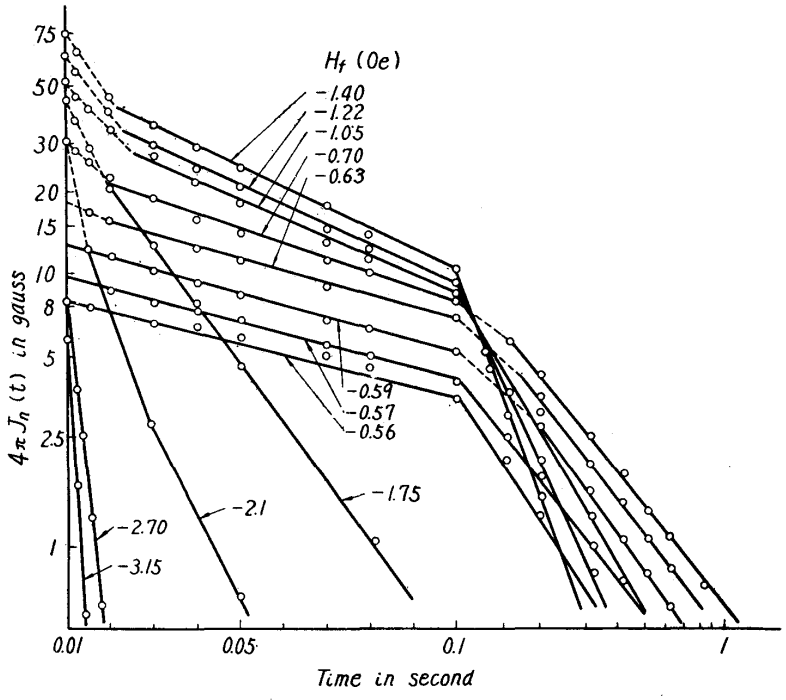

Fig. 4. Curves of $B_n(t)$vs. t for various H at $-195°$. $\log B_n(t)$ is replotted against t and $\log t$, using Maeda's data.

similar to that of Ni_3M_n, in which $\ln J_n(t)$ was not proportional to $\ln t$, but tended to decrease rather rapidly with the increase in $\ln t$. By replotting $\ln J_n(t)$ against t and $\ln t$ respectively, it was found that $\ln J_n(t)$ initially decreased in proportion to t up to 0.1 second and then became a linear function of $\ln t$. Consequently, $J_n(t)$ can be considered to consist of two parts, as shown in Fig. 4. Then, it may be inferred that the after-effect in iron is expressed by two theoretical formulas (14) and (15), both of which hold good under the conditions $t < t_s$ and $t > t_s$ respectively. t_s at the bend-point in the curve is about 0.1 second and will further be discussed below.

$\ln J_n(t)$ can be expressed, according as $t \gtrless t_s$, as follows:

$$\ln J_n(t) = \ln J_{no} - \frac{Kt_s}{J_{no}} \ln \frac{t}{t_s} + \text{const.},$$

and

$$\ln J_n(t) = \ln J_{no} - \frac{K}{J_{no}} t + \text{const}.$$

From the tangents of the curve $\ln J_n(t)$, K and t_s can easily be obtained, as shown in Table 4, in which the experimental values at $-195°$ were taken. J_{no} is the values of abscissa at $t_0 = 0.01$ second, intersected by extrapolation of linear part of the curve $\ln J_n(t) - t$. The upward deviation in the curve would arise from such a factor as a retardation due to the eddy current in the specimen caused by a sudden change in magnetization. As shown in the table, t_s is about 0.1 second in all cases, which agrees with that estimated from the curve in Fig. 4.

Table 4

H (Oe)	K	n	t_s (sec)	q (cm^3)	$qJ_s H$
−1.22	14.8	1.58	0.11	$1.2 \cdot 10^{-16}$	$2.3 \cdot 10^{-13}$
−1.05	14.2	1.31	0.092	1.3	2.3
−0.70	11.0	1	0.092	2.5	3.0
−0.63	9.8	0.94	0.096	5.3	5.5
−0.59	9.5	1	0.105	6.6	6.5

IV. Discussion

If $\ln J_{no}$ is either negligible compared with $\ln (kT/qJ_s)$ or constant, Eq. (17) will be reduced to Eq. (2). In general, however, it is not small and varies with N. Consequently, Eq. (17) can be said to be a general formula.

To displace a magnetic wall over some obstacles, an activation energy must be supplied by the field. Suppose a magnetic wall makes a stop against an obstacle where the critical field H_c is slightly larger than the strength of external field. The probability that the magnetic wall will move by the aid of thermal energy will be given by

$$p = a \exp\left\{ - \frac{q(H_c - H)J_s}{kT} \right\},$$

where a is the frequency factor of the fluctuation proportional to the frequency of vibration of the volume q, the number of elements in unit volume and entropy

change accompanied by the fluctuation. Consequently, a should be proportional to the number of walls activated by the field which, as mentioned already, will be proportional to the steepness of the magnetization curve. Since the slope of the magnetization curve in an irreversible region becomes gentle with the increase in the dimension ratio of the specimen, a or K in Eq. (3) decreases in inverse proportion to N, as can be seen in Table 1.

The linear relation between ΔJ and $\ln t$ leads to the conclusion that the activation energy increases in proportion to the logarithm of time and, in turn, the demagnetizing field varies linearly with the logarithm of time. Consequently, it will be certain that an internal field varying with time is superposed on the demagnetizing field in the course of after-effect. This conclusion has already been emphasized by Néel's[4] theory on the after-effect which is based on the internal perturbation of the demagnetizing field. According to his theory, the magnetic after-effect is classified into two kinds; one is due to the thermal fluctuation and the other is due to the diffusion of impurity atoms. The former is expressed by the equation similar to Eq. (13) and in the latter, the change in magnetization is expressed by the equation similar to Eq. (15). Thus, the fundamental equation (10) can explain two types of the after-effect simultaneously notwithstanding the present theory being based on a very simple assumption.

Further, Snoek[5] already pointed out that the elastic after-effect was analogous to the magnetic one in the reversible region, and concluded that the diffusion of interstitial atoms might be the main origin of both phenomena, as the activation energies estimated from the experimental results were the same in both cases. In the present case, the activation energy of wall displacement may be given by qJ_sH_{eff}. In the case of Ni_3M_n, q and H_{eff} are respectively 2.2×10^{-15}cm^3 and 0.6 Oe at 19°, and 6.1×10^{-17}cm^3 and 8.2 Oe at $-195°$ and, accordingly, the activation energy becomes about 0.6 eV at 19° and 0.2 eV at $-195°$; in the case of iron, it is about 0.2 eV at $-195°$, as shown in Table 4. Though the coincidence between the above activation eneries and those required for recovery of physical properties at these temperatures is fairly close[6], it seems difficult to explain the physical meaning of this activation energy, especially of the dependence of q on temperature, from the viewpoint of the magnetization process. It seems rather probable that the clue to the explanation of the mechanism of the after-effect may be found in the investgation on the interaction between the lattice imperfections and the magnetization process.

Summary

(1) The magnetic after-effect was assumed to be an activation-process.
(2) Taking account of the change in the effective field acting on the specimen, the theoretical formulas of the after-effect were deduced.

(4) L. F. Néel, Phys. Radium., **9** (1948), 184.
(5) J. G. Snoek, Physica, **8** (1939), 663.
(6) T. Broom, Adv. Phys., **3** (1954), 26.

(3) The theoretical formulas sufficiently expressed the characteristics of the magnetic after-effects observed in Ni_3M_n and technical iron.
(4) The activation energy estimated was about 0.2~0.6 eV, which suggested that the diffusion process was the main origin of the magnetic after-effect.
(5) The present theory was essentially equivalent to the Néel's theory of after-effect.

In conclusion, the present author wishes to express his hearty thanks to Prof. T. Sutoki for his helpful suggestions and encouragement throughout the work, and also to Mr. S. Maeda for his kind informations of valuable data.